Eddy Transport in Liquid-Metal Heat Transfer

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There are certain semiempirical equations for estimating heat transfer coefficients for liquid metals flowing through regular shapes. These contain the factor $\bar{\psi}$, which is the average, effective value of the ratio of the eddy diffusivity of heat transfer to that for momentum transfer. The main problem in liquid metal heat transfer has been to evaluate this quantity.

In the heating and cooling of turbulently flowing fluids, in a circular pipe, heat is transferred in the radial direction by two mechanisms: molecular conduction and eddy transport. In accordance with the classical mixing-length theory of Prandtl (*1*) small particles of fluid having fluctuating velocity components in the y direction, that is perpendicular to the wall, transport both sensible heat and momentum from their points of origin to their points of rest, or disappearance.

In liquid-metal heat transfer analysis the real problem has been to assess properly the eddy transport contribution in the total rate of heat transfer. In early theoretical studies the eddy diffusivity of heat transfer was either assumed equal to that for momentum transfer or simply treated as a parameter, whose values were unknown. Neither of these approaches led to theoretical relationships which could be relied upon to predict, at all times, liquid-metal heat transfer rates with sufficient accuracy.

For heat transfer to fluids flowing turbulently through symmetrical shapes such as pipes and annuli (as well as, in certain cases, through unbaffled rod bundles), it is possible to derive precise theoretical equations for expressing the heat transfer coefficient. For flow through pipes, under conditions of constant heat flux and fully-established turbulent flow, such an equation is (*2*)

$$\frac{1}{h} = \frac{4}{v_{av}^2 R^3} \int_R^0 \left[\int_r^R \frac{\int_0^r vrdr}{r\, k_{eff}}\, dr \right] vrdr \qquad (1)$$

where

$$k_{eff} = k + k_e = k + \psi \mu_e C_p$$

This work was done under the auspices of the U.S. Atomic Energy Commission.

But this equation has two drawbacks: it is unwieldly to use and it contains the variable quantity ψ, which cannot be evaluated with much certainty. The term ψ, defined as the ratio ϵ_H/ϵ_M, varies across the flow channel and with both Prandtl and Reynolds numbers.

To eliminate the first drawback in applying Equation (1) to liquid metals, Lyon found that it could be very closely approximated by the simple, semiempirical equation

$$N_{Nu} = 7.0 + 0.025(N_{Pe})^{0.8} \qquad (2)$$

assuming that ψ was constant and equal to 1.0. Lyon also found that if an average value of ψ, $\bar{\psi}$ were used in Equation (1), then Equation (2) became

$$N_{Nu} = 7.0 + 0.025(\bar{\psi} N_{Pe})^{0.8} \qquad (2a)$$

This equation has been known for several years, but the question of how to evaluate $\bar{\psi}$ in a particular case has never been satisfactorily settled. The use of $\bar{\psi}$ has the advantage over the use of ψ in that the problem of dealing with local variations in the latter is avoided. Thus with a reliable method of estimating $\bar{\psi}$, Equation (2*a*), in comparison with Equation (1), would be by far the more convenient means of estimating the heat transfer coefficient for turbulent flow of liquid metals in pipes.

There have been several attempts (*3, 4, 5, 6, 7, 8*) to produce a rational method of evaluating the contribution of eddy transport in liquid-metal heat transfer, but these have not wholly succeeded, thus far, in bringing theoretical predictions and experimental results, particularly the more recent results, into satisfactory agreement.

Figure 1 gives a comparison of several theoretical Nusselt number vs. Peclet number curves for predicting liquid-metal heat transfer rates. The Lyon-Martinelli curve (curve *A*) is based on the assumption of $\psi = 1$ at all Peclet numbers; the others are each based upon some rational method of evaluating ψ.

It would seem logical that the true Nusselt number vs. Peclet number curve, for turbulent flow and a given Prandtl number, should merge smoothly with the approximate molecular-conduction relationship $N_{Nu} = 7$ at the low-Peclet-number end and merge smoothly with the Lyon-Martinelli curve at high Peclet numbers. It is seen that none of the theoretical curves in Figure 1 appears to meet both of these conditions.

Molecular conduction remains the chief mechanism of heat transfer at the low end of the turbulent regime, because the eddies appear to lose essentially all of their heat while in transit. On the other hand when a Peclet number of about 5,000 is reached (for flow in circular tubes), which corresponds to a Reynolds number of about 250,000 for mercury and 700,000 for sodium, it would appear, on the basis of present information, that the eddies lose an insignificant fraction of their heat. But in contrast to the case of ordinary fluids, even under these conditions, an appreciable fraction of the total heat transfer may still be due to molecular conduction.

This paper presents yet another attempt to produce a relationship for evaluating the eddy diffusivity effect, but one which, in the opinion of the author, gives Nusselt number vs. Peclet number curves that are consistent with the above two principles and that give better agreement with recent, and presumably more accurate, experimental results. The derivation of this relationship, owing to the speculative nature of the subject, necessarily involves a certain degree of empiricism.

The purpose of this paper is to present a practical method of evaluating $\bar{\psi}$ for use in semiempirical equations, such as Equation (2*a*), to estimate liquid-metal heat transfer coefficients for turbulent flow in circular tubes, in annuli, and through rod bundles.

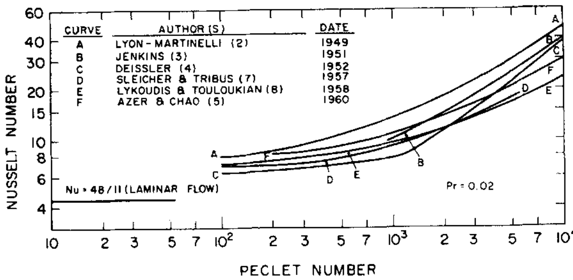

Fig. 1. Comparison of theoretical curves for representing liquid-metal heat transfer for flow in round tubes, under conditions of constant wall heat flux and fully-established flow.

THEORETICAL CONSIDERATIONS

In the following treatment the author shall consider the component of the eddy velocity in the mean flow, or x, direction, u', and that perpendicular to the mean flow, or y direction, v'. Next Prandtl's (*1*) mixing-length theory, which defines the mixing length by the equation

$$u' = l' \frac{du}{dy} \qquad (3)$$

where l' is the instantaneous mixing length, is employed. Then the eddy diffusivity of heat transfer is written as

$$\epsilon_H \equiv f_H l v' \qquad (4)$$

where the term f_H represents the ratio of the heat which the eddy actually transported through the distance l compared to what it could have transported if it did not lose any in transit. Since liquid metals have high molecular thermal conductivities, the eddies may lose a good portion of their heat to the surrounding liquid by both conduction and convection. The fraction lost $(1 - f_H)$ will of course vary with, among other things, the speed at which the eddy travels and the thermal conductivity of the liquid metal.

For fully-developed flow in a round tube the heat flux across an imaginary cylindrical surface, coaxial with the axis of the tube, by eddy conduction would be

$$q = f_H l v' C_p \rho \frac{dt}{dr} \qquad (5)$$

The lost flux at that particular location would then be

$$(1 - f_H) l v' C_p \rho \frac{dt}{dr} \qquad (6)$$

The rate of heat loss by an individual eddy would be

$$h(t_e - t)\,(\text{surface area of eddy}) \qquad (7)$$

where t_e and t are the temperatures of the eddy and surrounding fluid, respectively. It can be assumed that the ratio of total eddy surface to the imaginary cylindrical surface, which is related to the number of eddies per unit volume of fluid, is proportional to the eddy Reynolds number ϵ_M/ν raised to some power. Making this assumption one can combine Equations (6) and (7) to give

$$(1 - f_H) l v' C_p \rho \frac{dt}{dr} = (\text{a constant})\ h(t_e - t)\,(\epsilon_M/\nu)^n \qquad (8)$$

Drawing an analogy between velocity and temperature, as far as eddy properties are concerned, one can assume

$$t_e - t \propto l \frac{dt}{dr} \qquad (9)$$

Also one can write the equation

$$h = \frac{k}{b} \qquad (10)$$

where k is the molecular conductivity, and b represents the effective thickness of the film, around the eddy, through which the heat must be transferred. Equation (10) assumes that the eddy itself moves in streamline motion with respect to the surrounding medium. Streamline conditions were assumed, not because all the eddies move in this fashion, but because, for all practical purposes, it seems logical to assume that those which lose their heat are the ones which are moving more slowly and most probably in the laminar manner.

Combining Equations (8), (9), and (10) one obtains

$$(1 - f_H) C_p \rho v' = (\text{a constant})\ \frac{k}{b}\,(\epsilon_M/\nu)^n \qquad (11)$$

The definition of Nusselt number implies that the film thickness b is proportional to the particle diameter, and in turbulence the eddy diameter is generally assumed to be proportional to l.

Analogously to Equation (4) one now writes the eddy diffusivity for momentum transfer as

$$\epsilon_M \equiv f_M l v' \qquad (12)$$

With these relationships Equation (11) can be reduced to

$$f_H = \frac{1}{1 + \dfrac{a(\epsilon_M/\nu)^{n-1}}{(N_{Pr})\psi}} \qquad (13)$$

where a = a constant, and

$$\psi = \frac{\epsilon_H}{\epsilon_M} = \frac{f_H}{f_M} \qquad (14)$$

An analogous treatment will now be given for the eddy diffusivity of momentum transfer.

The shearing stress, due to momentum transfer by eddy diffusion, along an imaginary cylindrical surface at any radius in turbulent pipe flow is given by the equation

$$\tau = \frac{f_M l v' \rho}{g_o} \frac{du}{dr} \qquad (15)$$

The shear stress which is lost in transit is then

$$\frac{(1 - f_M) l v' \rho}{g_o} \frac{du}{dr} \qquad (16)$$

This can be related to the drag on the particle of fluid, which for viscous flow of the surrounding fluid is

$$F = \frac{3\pi d_o \mu v'}{g_o} \qquad (17)$$

The shearing stress on the particle becomes proportional to F/d_o^2, or to

$$\frac{3\pi\mu v'}{g_o d_o} \qquad (18)$$

Equations (16) and (18) can now be combined, and the term $(\epsilon_M/\nu)^n$ incorporated as in Equation (8), to give

$$1 - f_M = (\text{a constant})\ \frac{\mu}{\rho v' d_o}\,(\epsilon_M/\nu)^n \qquad (19)$$

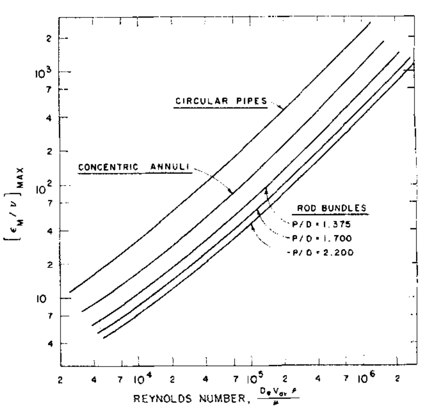

Fig. 2. Values of $[\epsilon_M/\nu]_{max}$ for fully established turbulent flow of liquid metals through circular tubes, annuli, and rod bundles with equilateral triangular spacing.

Remembering that the actual particle Reynolds number

$$\frac{d_o v' \rho}{\mu} = (\text{a constant}) \frac{\epsilon_M/\nu}{f_M} \quad (20)$$

one can reduce Equation (19) to

$$f_M = \frac{1}{1 + c(\epsilon_M/\nu)^{n-1}} \quad (21)$$

where c equals a constant. Finally substituting Equations (13) and (21) into (14), and simplifying, one obtains

$$\psi = 1 - \frac{(a/N_{Pr}) - c}{(\epsilon_M/\nu)^m} \quad (22)$$

There are insufficient experimental results available to evaluate all three constants, a, c, and m, in this equation, but there is good reason to believe, as explained later, that c is small compared with the a/N_{Pr} term and can be neglected. Therefore

$$\bar{\psi} = 1 - \frac{a}{N_{Pr}} [(\nu/\epsilon_M)^m]_{av} \quad (22a)$$

where both ψ and $(\nu/\epsilon_M)^m$ are averaged over the channel area. Evaluating a and m in this equation would involve a lot of trial-and-error calculations. Moreover Equation (22a) would not be a very convenient equation to use anyway for calculating values of $\bar{\psi}$.

A more convenient approach is to write the equation

$$\bar{\psi} = 1 - \frac{a}{N_{Pr}(\epsilon_M/\nu)^m_{max}} \quad (22b)$$

and let the constants readjust to maintain the applicability of the equation. This is feasible because $(\nu/\epsilon_M)_{av}$ and $(\epsilon_M/\nu)_{max}$ bear a fixed and equal ratio to each other for flow through pipes and annuli and for in-line flow through rod bundles. Since the quantity $(\epsilon_M/\nu)_{max}$ is relatively easy to obtain, Equation (22b) is convenient to use in evaluating $\bar{\psi}$ for use in semiempirical equations, such as (2a), for estimating heat transfer coefficients.

Values of $[\epsilon_M/\nu]_{max}$ used in this paper are shown plotted against the Reynolds number in Figure 2.

For flow in pipes values of $[\epsilon_M/\nu]_{max}$ were calculated from the following equation (4):

$$\epsilon_M/\nu = 0.72S^*\left(1 - \frac{y}{R}\right)(1 - \sqrt{1 - y/R}) \quad (23)$$

where $S^* = (R/\nu)\sqrt{\tau g_o/\rho}$. The quantity ϵ_M/ν reaches a maximum at $y/R = 5/9$.

For flow in annuli ϵ_M/ν was obtained by first calculating ϵ_M by use of the standard shear-stress equation

$$\frac{\tau g_o}{\rho} = -\left[\frac{\mu}{\rho} + \epsilon_M\right]\frac{du}{dr} \quad (24)$$

The velocity distribution results of Rothfus (9) were used, and the method of calculation was the same as that used by Dwyer and Tu (*10*). For a given set of conditions the value of $(\epsilon_M/\nu)_{max}$ for the flow in that portion of an annulus lying between r_1 and r_m is the same as that for the flow in that portion lying between r_m and r_2.

For the case of in-line flow through rod bundles having equilateral triangular spacing ϵ_M was also calculated by means of Equation (24) with the annulus velocity-profile results of Rothfus and the method of Dwyer and Tu.

With reference to Figure 2 it will be noticed that, for the various geometries, there is a large spread in the values of $[\epsilon_M/\nu]_{max}$ for a given Reynolds number, with pipes giving the highest values and bundles the lowest. This means that for a given Reynolds number, or for a given Peclet number (at constant Prandtl number), values of $\bar{\psi}$ are greatest for flow in pipes and least for flow through bundles. Thus in the latter case considerable error can be introduced by assuming that $\bar{\psi}$ equals unity, as is often done for flow in pipes, particularly at the lower Peclet numbers.

The two constants, a and m, in Equation (22b) must be evaluated from experimental data. Theoretically they can be evaluated from experimental determinations of either ψ or h.

There are available three sets of results on the experimental determination of ψ, which might be used to evaluate a and m. They, shown in Figure 3, were all obtained on mercury, were based on measurement of velocity and temperature profiles, and were supposed to be obtained under conditions of turbulent flow where both velocity and temperature profiles were fully established. Isakoff and Drew (*21*) made their measurements in a 1.5 in. I.D. stainless steel electrically-heated round tube in the Reynolds number range 36,700 to 373,000. Later Brown et al. (*11*) made measurements in a 1.61 in. I.D. nickel steam-heated round tube in the Reynolds number range 250,000 to 800,000. And finally Mizushina and Sasano (*6*) made measurements in a 25 × 150 mm. stainless steel rectangular channel in the Reynolds number range 10,000 to 130,000. The channel was heated on one side and cooled on the other by flowing water.

TABLE 1. CRITICAL PECLET NUMBERS

N_{Pr}	Tubes	Annuli*	Δ rod bundles $P/D = 1.375$	$P/D = 1.700$	$P/D = 2.200$
0.005	117	270	460	622	770
0.01	131	300	530	720	890
0.02	144	330	582	800	1,000
0.03	150	345	603	840	1,056

* For heat transfer through either the inner or outer wall.

It is immediately apparent from Figure 3 that the agreement between the three sets of results is poor. Presumably it is the difficulty in making highly accurate temperature measurements which is mostly responsible for the lack of agreement, for it is not the temperatures but the temperature gradients which are important. The quantity $\bar{\psi}$ in the figure represents the average value of Ψ, as defined by the equation

$$\bar{\psi} = 2\int_0^1 \psi(r/R)d(r/R) \quad (25)$$

for flow in circular tubes.

Owing to the great differences which exist between the ψ determinations of the different investigators, it was decided that it would be better to evaluate the constants in Equation (22b) from h measurements, instead. These measurements were by far the most numerous for $Pr = 0.02$. Under these conditions Equation (22b), after the constants are evaluated, becomes

$$\bar{\psi} = \left[1 - \frac{91.0}{(\epsilon_M/\nu)^{1.4}_{max}}\right]_{N_{Pr}=0.02} \quad (22c)$$

This equation is shown plotted in Figure 4. For a given Peclet number it appears that on the basis of existing results $\bar{\psi}$ is essentially independent of Prandtl number in the fully turbulent regime. This is tantamount to saying that the constant c in Equation (22) is negligible compared with the a/N_{Pr} term. Under these conditions Equation (22c) becomes

$$\bar{\psi} = 1 - \frac{1.82}{N_{Pr}(\epsilon_M/\nu)^{1.4}_{max}} \quad (22d)$$

which was then used to calculate the curves for $N_{Pr} = 0.01$ and 0.03 in Figure 4.

With reference to Equation (22d), for a given value of the Prandtl number, as the flow rate is reduced, a value of the Reynolds number will eventually be reached at which $\bar{\psi}$ becomes zero. Below this Reynolds number, and corresponding Peclet number, the equation no longer holds. A listing of these critical Peclet numbers, for the geometries considered in this paper, is given in Table 1. They were obtained by solving Equation (22*d*) after setting $\bar{\psi}$ equal to zero.

As the Peclet number is increased, the critical value is that at which eddy transport begins to assert itself in contributing to the total convective heat transfer rate.

Values of the parameter $\bar{\psi}$, evaluated by Equation (22*d*), will now be used to predict new N_{Nu} vs. N_{Pe} curves for turbulent flow of liquid metals through circular tubes, annuli, and rod bundles. These curves will then be compared with experimental results.

In choosing the experimental results for comparison with the new theoretical curves, no results published prior to 1956 were used. Before that date results were for the most part obtained on mercury and lead-bismuth alloy, under conditions where wetting probably did not exist. This meant that entrained gas and/or particulate matter, which are often present, probably collected at the interface between the flowing metal and heated surface, contributing to low results. Results obtained in recent years, under improved experimental conditions, tend to give higher coefficients, particularly at the higher Peclet numbers.

In the comparisons below the experimental results which are used were obtained, for the most part, under wetting conditions or where relatively high contact resistances were not suspected.

FLOW IN CIRCULAR TUBES

Equation (2*a*) is the standard semi-theoretical, semiempirical equation for estimating heat transfer rates to liquid metals flowing in circular tubes under conditions of constant heat flux and fully-established flow. In this equation the parameter $\bar{\psi}$ is often taken as unity at all values of the Peclet number. Combining Equations (2*a*) and (22*d*) one gets

$$N_{Nu} = 7.0 + 0.025\left[N_{Pe} - \frac{1.82\, N_{Re}}{(\epsilon_M/\nu)^{1.4}_{\max}}\right]^{0.8} \qquad (26)$$

which will now be tested against experimental results. It applies when the Peclet number is above the critical value, that is above the highest Reynolds number, for a given Prandtl number, at which molecular conduction is the sole mode of heat transfer.

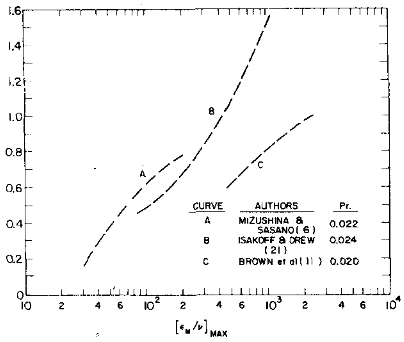

Fig. 3. Comparison of the results of the experimental determination of the eddy diffusivity ratio by various investigators, showing the effect of turbulence on $\bar{\psi}$.

High Peclet Number Range

Figure 5 shows the experimental results of Brown et al. (*11*) and Kirillov et al. (*12*), both for mercury flowing in nickel tubes. For this case, particularly in this range of Peclet numbers, there is little difference between Equations (2) and (26). Nevertheless Equation (26) appears to fit the data points better. The two sets of experimental results are in good agreement. The Hg-Ni system is a wetting system. Therefore there should have been no contact resistances to heat transfer between solid and liquid metals.

Intermediate Peclet Number Range

In this range three sets of experimental data are available for comparison with Equations (2) and (26). See Figure 6. Owing to lack of agreement between the results of the different investigators, a clearcut comparison between theory and experiment is not possible.

The experimental results of Kirillov et al. (*12*), and Khabakhpaseva and Il'in (*13*), below $Pe = 300$, agree quite well with the recommended curve. Above 300 they agree very well with the Lyon-Martinelli curve. The results of Novikov et al. (*14*) are judged to be definitely too low. Again all three investigations were conducted with wetting systems. The recommended curve appears to represent a compromise between the differing results.

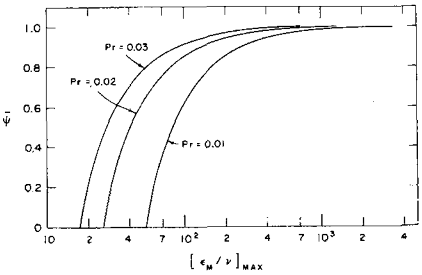

Fig. 4. Graphical representation of Equation (22*d*).

Low Peclet Number Range

Calculated Nusselt numbers from Equations (2) and (26) will now be compared with the experimental results of Pirogov (*15*) for flow of sodium in 20-mm. I.D. copper tubes. The data were apparently taken under conditions of constant heat flux and fully-established flow. As seen in Figure 7, at the low Peclet numbers, the results agreed with the condition that $Nu = 7.0$, as they should, but as the flow rate was increased, they fell more and more above the Lyon-Martinelli curve. Thus Pirogov's results, at the highest flow rates, are undoubtedly too high and therefore give poor agreement with Equation (26).

There is one other set of experimental results obtained in this range. Petukhov and Yushin (*16*) obtained them for flow of mercury through a carbon-steel tube. They covered both the laminar- and turbulent-flow regimes. In the laminar regime the results agreed with the equation $Nu = 48/11$, as they should, but in the turbulent regime they are believed to be quite low. The validity of these results is clouded by the fact that wetting in all probability did not exist. For this reason the results were not shown in Figure 7.

Recommended Curves

Figure 8 shows the suggested curves for representing heat transfer conditions to liquid metals flowing in a circular tube under conditions of constant heat flux and fully-established flow. In the laminar region $N_{Nu} = 48/11$; in the lower turbulent region, where molecular conduction is the only mode of heat transfer, $N_{Nu} = 7.0$; in the middle and upper turbulent regions, where both molecular conduction and eddy conduction exist, N_{Nu} is given by Equation (26).

For flow in tubes it is apparent that the recommended curves are, after all, not greatly different from the Lyon-Martinelli curve. At both ends of the turbulent-flow regime there is perfect agreement. The greatest disagreement occurs in the region of transition from total to partial heat transfer by molecular conduction. The transition point varies slightly, depending on Prandtl number. At these points the recommended N_{Nu} vs. N_{Pe} curve gives Nusselt numbers which are about 14% below that by Equation (2).

In Figure 8 the dashed lines represent the possible range of Reynolds numbers encountered in passing between laminar- and turbulent-flow regimes.

FLOW IN A CONCENTRIC ANNULUS

Application of Equation (22*d*) to annuli will be compared with two sets of typical experimental data, one for the case of heat transfer through the inner wall only and one for the case of heat transfer through the outer wall only. In both cases the experimental coefficients were calculated from surface temperature measurements taken under conditions of constant heat flux and fully-established flow.

Heat transfer to liquid metals flowing through a concentric annulus under conditions of constant heat flux, fully-established turbulent flow, and heat transfer through the inner wall only, is represented by the following semiempirical equation of Dwyer and Tu (*17*):

$$N_{Nu,\,1} = \alpha_1 + \beta_1(\bar{\psi}N_{Pe})^{\gamma_1} \qquad (28)$$

where

$$\alpha_1 = 4.63 + 0.686y$$

$$\beta_1 = 0.02154 - 0.000043y$$

and

$$\gamma_1 = 0.752 + 0.01657y - 0.000883y^2$$

This equation corresponds to Equation (2*a*) for flow inside circular tubes. It will now be compared with experimental results both ways, that is where $\bar{\psi}$ is taken as unity and where it is evaluated by Equation (22*d*). Figure 9 shows the results of Petrovichev (*18*) for flow of mercury through a steel annulus having a y value of 1.55. The points fall in the upper Peclet number range, where there happens to be little difference between the recommended curve and that of Equation (28). Nevertheless the agreement between the experimental results and the recommended curve is not bad.

For heat transfer through the outer wall only, the semiempirical equation, by Dwyer and Tu, is

$$N_{Nu,\,2} = \alpha_2 + \beta_2(\bar{\psi}N_{Pe})^{\gamma_2} \qquad (29)$$

where

$$\alpha_2 = 5.26 + 0.0500y$$

$$\beta_2 = 0.01848 + 0.003154y - 0.0001333y^2$$

and

$$\gamma_2 = 0.780 - 0.01333y + 0.000833y^2$$

This equation, with $\bar{\psi}$ taken as unity at all Peclet numbers and with $\bar{\psi}$ evaluated in accordance with Equation (22*d*), is compared in Figure 10 with the experimental results of Petrovichev (*18*) for cooling mercury in a steel annulus having a y value of 1.67. Again the agreement between the experimental results and the recommended curve is not bad, the former falling just slightly below the latter. Wetting conditions presumably did not exist, and therefore there was the possibility of some contact resistance.

IN-LINE FLOW THROUGH ROD BUNDLES

There are two semiempirical equations in the literature for this case, both based upon the conditions of fully-established turbulent flow, constant heat flux, and equilateral triangular pitch. Dwyer and Tu's (*10*) equation is

$$N_{Nu} = 0.93 + 10.81(P/D) - 2.01(P/D)^2 + 0.0252(P/D)^{0.273}(\bar{\psi}N_{Pe})^{0.8} \qquad (30)$$

which was developed for the ranges $10^2 \leq Pe \leq 10^4$ and $1.3 \leq P/D \leq 2.5$, while Friedland and Bonilla's (*24*) equation is

$$N_{Nu} = 7.0 + 3.8(P/D)^{1.52} + 0.027(P/D)^{0.27}(\bar{\psi}N_{Pe})^{0.8} \qquad (30a)$$

which applies to the ranges $0 \leq Pe \leq 10^5$ and $1.3 \leq P/D \leq 10$. In the upper turbulent region, where eddy conduction is a contributing mode of heat transfer, Equation (30) is believed to be more reliable than (30*a*), owing to the fact that it was based on more complete, more recent, and presumably more accurate velocity-profile data than was Equation (30*a*).

With the same procedure as before Equation (30) is plotted in Figure 11 two ways: with $\bar{\psi}$ assumed equal to unity and with $\bar{\psi}$ evaluated in accordance with Equation (22*d*). The agreement between the former curve and the experimental results of Friedland et al. (*19*) is very poor, whereas that between the latter curve and the experimental results is excellent. The experimental points shown in Figure 11 are the same as those given in Figure 5 of reference (*19*), except that slight corrections for variation in heat loss from the test section have been applied. The results were obtained on the central rod of a thirteen-rod bundle. It was chromium-plated and therefore unwetted by the mercury. The results shown are believed to be reliable for they have since been duplicated several times (*21*) by results on other elements, both wetted and unwetted, at the Brookhaven National Laboratory. For perfectly clean and gas-free systems there is, in the present author's opinion, no difference between wetted and unwetted liquid-metal heat transfer results.

It will be noticed that the difference between the two curves in Figure 11 is quite large compared with the difference between the two curves in Figure 5, for example. This is due to the fact that, at a given Reynolds number, the value of $(\epsilon_M/\nu)_{max}$ is much less for flow through rod bundles than for flow through pipes. This leads to a situation in the lower portion of the turbulent flow regime which is of considerable interest, as far as heat transfer to liquid metals flowing in-line through rod bundles is concerned. Figure 12 presents what the author be-

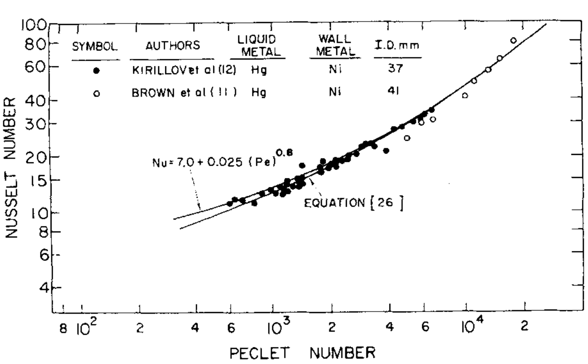

Fig. 5. Heat transfer to mercury flowing in round tubes in the high Peclet number range, under condition of constant heat flux. Comparison between semiempirical equations and experimental results.

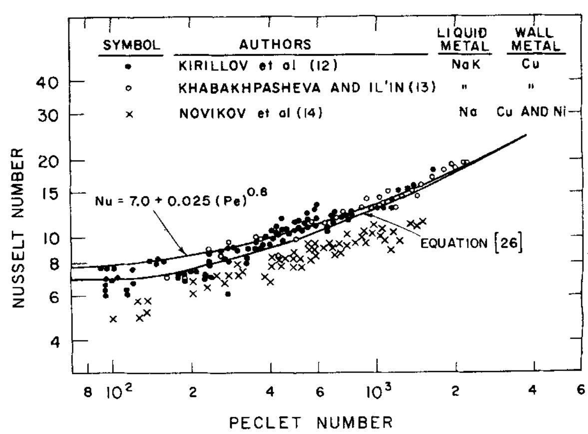

Fig. 6. Heat transfer to liquid metals flowing in round tubes in the intermediate Peclet number range under condition of constant heat flux. Comparison between semiempirical equations and experimental results.

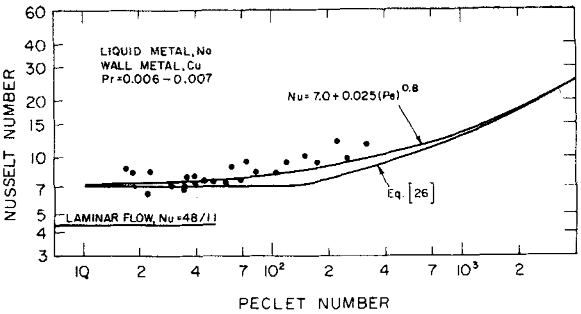

Fig. 7. Heat transfer to liquid metals flowing in round tubes in the low Peclet number range under condition of constant heat flux. Comparison between semiempirical equations and experimental results.

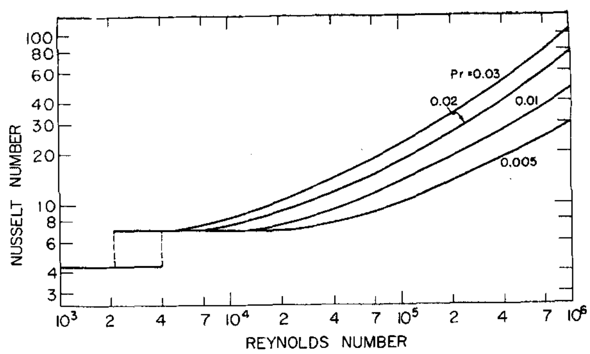

Fig. 8. Suggested curves for heat transfer to liquid metals flowing through circular tubes under conditions of constant heat flux and fully-established flow.

lieves to be the situation for heat transfer to rod bundles, in the turbulent-flow regime, from barely turbulent to very turbulent flow. Curves are shown for three different P/D ratios. The interesting thing is that the Nusselt number, for a given P/D ratio, remains essentially constant for a long distance into the turbulent regime, indicating that in this region the eddy contribution to heat transfer is negligible. As the flow rate is increased, a point is reached where eddy transfer begins rapidly to assert itself, and soon thereafter the Nusselt number increases almost proportionately with the 0.5 power of the Peclet number.

Over the lower end of the turbulent flow regime, where eddy transport is insignificant, it is estimated that the Nusselt number is not absolutely constant but increases slightly (~5%) owing to the changing velocity profile with increase in flow.

It is now apparent that many liquid-metal heat exchangers have been designed to operate under conditions where the heat transfer coefficients for in-line flow through the tube bundles are not significantly higher than they would be if streamline flow conditions existed.

In the case of flow in tubes it is estimated that for $N_{Pr} = 0.02$ the Nusselt number begins to rise above the molecular-conduction value at a Peclet number of around 145, while in the case of in-line flow through bundles the corresponding Peclet number is about 580 at a P/D ratio of 1.38 and about 1,000 at a P/D ratio of 2.20. Again the reason for this is found in Figure 2; that is for a given Reynolds number values of $[\epsilon_M/\nu]_{max}$ for pipes are about a factor of 4 greater than those for bundles. In other words for bundles for example below $N_{Re} = 50{,}000$ (for $P/D = 2.20$ and $N_{Pr} = 0.02$) the eddies do not travel fast enough to prevent losing essentially all their sensible heat while in transit.

ALTERNATIVE EQUATIONS FOR $\bar{\psi}$

By making somewhat different assumptions it is possible to derive a number of equations, besides (22), for evaluating the quantity ψ, all based upon the basic idea used in deriving Equation (22).

For example if the $(\epsilon_M/\nu)^n$ term is omitted from Equations (8) and (11), which is simply to assume that the heat and momentum undelivered by the eddy particle is proportional to the heat and momentum respectively lost by a representative single particle in transit, then

$$\psi = 1 - \frac{(\text{a constant}) + \dfrac{\text{a constant}}{N_{Pr}}}{\epsilon_M/\nu} \qquad (31)$$

which is the same as Equation (22), except there is no exponent on the ϵ_M/ν term.

Again if in deriving Equation (31) one had assumed that ϵ_M were proportional to bv', one would have obtained

$$\psi = \frac{1 - \dfrac{\text{a constant}}{N_{Pr}(\epsilon_M/\nu)}}{1 - \dfrac{\text{a constant}}{\epsilon_M/\nu}} \qquad (32)$$

If one had assumed a model where the flow was turbulent rather than streamline, instead of Equation (10) for h and (17) for F, one would have to use their turbulent counterparts. For h one can use the following equation, proposed by Ranz and Marshall (23):

$$\frac{hd_o}{k} = 2.0 + 0.6(d_o v'/\nu)^{1/2}(C_p\mu/k)^{1/3} \qquad (33)$$

and for F the equation would be

$$F = \frac{(\text{a constant})\rho(v')^2 d_o^2}{g_o} \qquad (34)$$

Using these equations, and the same general procedure employed in deriving Equation (22), one can obtain a comparable relationship for ψ, on the basis of a turbulent model. This relationship however is complex. It can be simplified by making assumptions similar to those made in obtaining Equations (31) and (32). For example if it is assumed that the particle Reynolds number is proportional to ϵ_M/ν, one obtains

$$\psi = \frac{1 - \dfrac{(\text{a constant})(\epsilon_M/\nu)^{n-1}}{N_{Pr}} - \dfrac{(\text{a constant})(\epsilon_M/\nu)^{n+0.5}}{N_{Pr}^{2/3}}}{1 - (\text{a constant})(\epsilon_M/\nu)^n} \qquad (35)$$

This equation, having four constants, could be made to fit almost any curve, and therefore it does not prove very much. Making the further simplifying assumption of dropping the $(\epsilon_M/\nu)^n$ term from Equation (8), as was done to obtain Equation (31), one ends up with

$$\bar{\psi} = (\text{a constant}) - \frac{(\text{a constant}) - (\text{a constant})N_{Pr}^{1/2}[\epsilon_M/\nu]^{1/3}}{N_{Pr}[\epsilon_M/\nu]} \qquad (36)$$

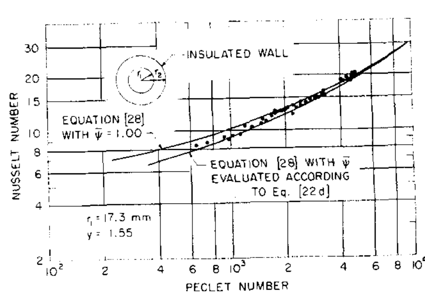

Fig. 9. Heat transfer rates to mercury flowing in an annulus under conditions of constant heat flux, fully-established flow, and heat transfer through inner wall only. Comparison between semiempirical equations and experimental results of Petrovichev (18).

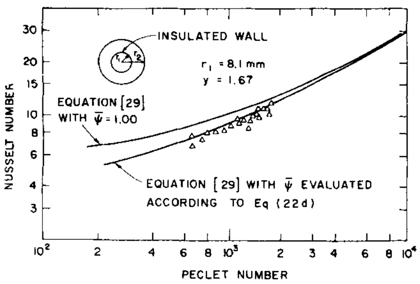

Fig. 10. Heat transfer rates to mercury flowing in a steel annulus under conditions of constant heat flux, fully-established flow, and heat transfer through outer wall only. Comparison between semiempirical equations and experimental results of Petrovichev (*18*).

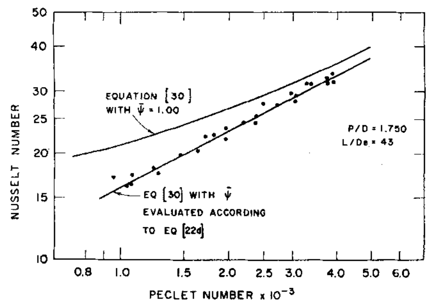

Fig. 11. Heat transfer to mercury flowing in-line through unbaffled rod bundle under conditions of constant heat flux and fully-established flow. Comparison between semiempirical equations and experimental results of Friedland et al. (*19*) for mercury.

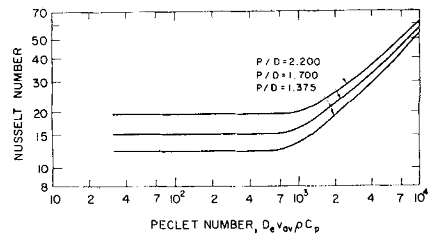

Fig. 12. Recommended curves for predicting heat transfer rates to liquid metals flowing in-line through unbaffled rod bundles under conditions of constant heat flux and fully-established flow.

It was subsequently found that the first constant in this equation was so close to unity that it was set at that, the two constants in the numerator then being evaluated from experimental data.

Equation (31) actually has two constants, but one of them was dropped as was done in Equation (22), for two reasons. First, there are not sufficient experimental results available for different values of the Prandtl number to evaluate the second constant. Second, this constant is most probably negligible compared with the other term in the numerator of the fraction. This latter assumption is consistent with the idea that in the higher part of the Peclet number range the Nusselt number is independent of Prandtl number for liquid metals.

After one evaluates the constants in Equations (31), (32), and (36) from experimental results, as was done in the case of Equation (22*d*), the equations finally become

$$\bar\psi = 1 - \frac{0.36}{N_{Pr}[\epsilon_M/\nu]_{\max}} \qquad (31a)$$

$$\bar\psi = \frac{1 - \dfrac{0.524}{N_{Pr}[\epsilon_M/\nu]_{\max}}}{1 - \dfrac{12.75}{[\epsilon_M/\nu]_{\max}}} \qquad (32a)$$

and

$$\bar\psi = 1 - \frac{0.700 - 0.157 N_{Pr}^{1/2}[\epsilon_M/\nu]^{1/3}_{\max}}{N_{Pr}[\epsilon_M/\nu]_{\max}} \qquad (36a)$$

As before $\bar\psi$ and $[\epsilon_M/\nu]_{\max}$ were used instead of ψ and ϵ_M/ν when the constants were determined. These three equations give $\bar\psi$ vs. $[\epsilon_M/\nu]_{\max}$ curves very similar to those for Equation (22*d*) shown in Figure 4. This is so, even for Equation (31*a*) which has only a single constant. The similarity of these equations, as far as their curves are concerned, shows that any of them could be used to evaluate $\bar\psi$ without causing great displacement in the N_{Nu} vs. N_{Pe} curve, which tends to give added support to the basic analytical model which has been used.

Another point is worthy of mention. Equations (32*a*) and (36*a*) give values of $\bar\psi$ which indicate the Nusselt number to be independent of the Prandtl number in the higher Peclet number range. This was, in part, the justification for dropping one of the two constants in Equations (22) and (31).

Equation (36*a*), with respect to the other three equations, is unique in that it gives values of $\bar\psi$ which can exceed unity. There is no theoretical reason why $\bar\psi$ cannot exceed unity. In fact this author suspects that it may well do so, at very high Reynolds numbers. This then raises a question concerning the assumption of streamline flow conditions at the boundary layers surrounding the individual eddy particles, which was made in deriving Equation (22). It may well be that this assumption is not valid at very high degrees of turbulence. If so an equation such as (22*d*) would be the one to use in the lower portion of the turbulent regime and one such as (36*a*) in the upper portion. In any event the departures from unity would probably be very small and therefore have little effect on the N_{Nu} vs. N_{Pe} curve. Before such fine distinctions are possible, more experimental results of high accuracy are required.

CONCLUSIONS

A very practical method has been presented for evaluating the quantity $\bar\psi$ for use in semiempirical equations for estimating heat transfer coefficients for turbulent, internal flow of liquid metals.

Owing to the great difficulty in getting accurate liquid metal heat transfer data, it should be expected that there will be some disagreements with any type of correlation, no matter how accurate. When one considers this, the applicability of Equation (22*d*) to flow of liquid metals through tubes, annuli, and rod bundles has been quite well established. The fact that $\bar\psi$, for use in Equations (2*a*), (28), (29), and (30), can be satisfactorily evaluated by a single relationship is taken as an indication of its fundamental soundness.

Based upon the present study the following procedures are therefore recommended for estimating heat transfer rates for liquid metals flowing under conditions where the heat flux is constant, and the flow is turbulent and fully established.

Flow in a circular tube
- Below critical Peclet number, $N_{Nu,\,mc} = 7.0$
- Above critical Peclet number, Equation (26)

Flow in a concentric annulus with heat transfer from inner wall only
- Below critical Peclet number

$$N_{Nu,\,mc,\,1} = 4.92 + 0.686y \qquad (37)$$

- Above critical Peclet number, Equations (22*d*) and (28)

Flow in a concentric annulus, with heat transfer from outer wall only
- Below critical Peclet number

$$N_{Nu,\,mc,\,2} = 5.52 + 0.076y \qquad (38)$$

- Above critical Peclet number, Equations (22*d*) and (29)

In-line flow through a rod bundle (equilateral triangular spacing, and 1.3 > *P*/*D* > 5
- Below critical Peclet number

$$N_{Nu,\,mc} = 3.65 + 5.75(P/D)^{1.27} \qquad (39)$$

- Above critical Peclet number, Equations (22*d*) and (30)

Equation (39) was obtained from

calculated results at $P/D = 1.7$, with the velocity profile of Rothfus et al. (9) and the assumption that the ratio $(N_{Nu,\,mc} - N_{Nu,\,L})/(N_{Nu,\,S} - N_{Nu,\,L})$ was independent of the P/D ratio. In the Peclet number range 1.3 to 5.0, Nusselt numbers by Equation (39) turn out to be less than 5% less than those calculated from the first two terms in Equation (30*a*). To arrive at Equation (39) the calculated laminar results of Sparrow et al. (22) and the slug-flow results of Friedland and Bonilla (24) were used.

The constants in Equations (22*d*), (31*a*), (32*a*), and (36*a*) have been evaluated with what are considered as the most dependable experimental results available at this time. As additional results become available, these constants can be expected to change.

NOTATION

a = a constant in Equation (13)
b = effective thickness of film around eddy particle, ft.
c = constant in Equation (21)
C_p = specific heat, (B.t.u./lb.-mass) (°F.)
d_e = diameter of eddy particle, ft.
D = outside diameter of rod or tube, ft.
D_e = four (cross-sectional flow area)/wetted perimeter, ft.
f_H = fraction of heat energy, originally possessed by an eddy particle, which is delivered at point where particle comes to rest with respect to rest of fluid
f_M = same as f_H except for momentum transfer
F = drag force on eddy particle, lb.-force
g_o = conversion factor, (lb.-mass) (ft.)/(lb.-force)(hr.)^2
h = heat transfer coefficient, B.t.u./(hr.)(ft.)^2(°F.)
k = thermal conductivity, B.t.u./(hr.)(ft.)(°F.)
l = average eddy mixing length, ft.
l' = instantaneous eddy mixing length, defined by Equation (3), ft.
m = exponent in Equation (22)
n = exponent in Equation (8)
N_{Nu} = hD_e/k = Nusselt number, dimensionless
$N_{Nu,mc}$ = Nusselt number for turbulent flow involving molecular conduction only
$N_{Nu,1}$ = Nusselt number for heat transfer to liquid metals flowing through annuli, where heat is transferred from the inner wall only, dimensionless
$N_{Nu,mc,1}$ = same as $N_{Nu,1}$ except it involves molecular conduction only
$N_{Nu,2}$ = Nusselt number for heat transfer to liquid metals flowing through annuli, where heat is transferred from the outer wall only, dimensionless
$N_{Nu,mc,2}$ = same as $N_{Nu,2}$ except it involves molecular conduction only
$N_{Nu,L}$ = laminar-flow Nusselt number
$N_{Nu,S}$ = slug-flow Nusselt number
P = rod pitch, or distance between rod centers in bundle, ft.
N_{Pe} = $\dfrac{D_e v_{av} \rho C_p}{k}$ = Peclet number, dimensionless
N_{Pr} = $C_p\mu/k$ = Prandtl number, dimensionless
q = heat flux, B.t.u./(hr.)(ft.)^2
r = radial distance, ft.
r_1 = inner radius of annulus, ft.
r_2 = outer radius of annulus, ft.
r_m = radius of maximum velocity in concentric annulus, ft.
R = radius of circular tube, ft.
N_{Re} = $\dfrac{D_e v_a \rho}{\mu}$ = Reynolds number, dimensionless
S^* = $(R/\nu)\sqrt{\tau g_o/\rho}$ = tube-radius parameter, dimensionless
t = temperature, °F.
t_e = temperature of eddy particle, °F.
u = linear velocity at radius r in x direction, ft./hr.
u' = fluctuating component of velocity in axial direction, ft./hr.
v' = same as u' except in y direction, ft./hr.
v_{av} = average linear velocity through pipe, annulus, or rod bundle, ft./hr.
x = axial distance, ft.
y = radial distance measured from tube wall, ft.; also, with reference to concentric annuli, $y = r_2/r_1$, dimensionless

Greek Letters

$\alpha_1, \beta_1, \gamma_1$ = quantities defined in Equation (28), dimensionless
$\alpha_2, \beta_2, \gamma_2$ = quantities defined in Equation (29), dimensionless
ϵ_H = eddy diffusivity for heat transfer, sq.ft./hr.
ϵ_M = eddy diffusivity for momentum transfer, sq.ft./hr.
μ = molecular viscosity, lb.-mass/(ft.)(hr.)
μ_e = eddy viscosity, lb-mass/(ft.)(hr.)
ν = kinematic viscosity, sq.ft./hr.
ρ = density, lb.-mass/cu.ft.
τ = shear stress at radius r, lb.-force/sq.ft.
ψ = ϵ_H/ϵ_M, dimensionless
$\bar{\psi}$ = average value of ψ for use in Equations (2*a*), (28), (29), (30), and (30*a*), dimensionless

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Manuscript received March 21, 1962; revision received September 19, 1962; paper accepted October 30, 1962.